# International Conference
# «New Trends in Mathematical and Theoretical Physics»

Book of abstracts

Steklov Mathematical Institute

Moscow, October 3-7, 2016

# Новые направления в математической и теоретической физике*

## New trends in quantum probability and quantum mechanics

Luigi Accardi
*Centro Vito Volterra - University of Rome Torvergata*

(October 3, 2016, 10:00)

The solution of the problem to generalize, to the multi-dimensional case, the Favard Lemma, which lies at the foundations of the theory of orthogonal polynomials in one real variable, was recently obtained in the joint paper [1]. This suggests a new approach to the theory of orthogonal polynomials, that emphasizes the fact that it generalizes in several, unexpected and non-trivial ways the usual mathematical structure of quantum mechanics and (in its infinite dimensional version) quantum field theory.

These generalizations will be briefly described and it will explained in what sense they extend the program of "non-linear quantization" whose first achievement was obtained in the paper [2].

## Список литературы

[1] Accardi L., Barhoumi A., Dhahri A., "Identification of the theory of orthogonal polynomials in $d$–indeterminates with the theory of 3–diagonal symmetric interacting Fock spaces on $\mathbb{C}_d$", submitted to IDA-QP.

[2] Accardi L., Lu Y.G., Volovich I.V.:, "White noise approach to classical and quantum stochastic calculi", Lecture Notes of the Volterra–CIRM International School with the same title, Trento, Italy, 1999, Volterra Preprint N. 375, July (1999).

## Holographic quantum photosynthesis and black holes

Igor Volovich
*Steklov Mathematical Institute*

(October 3, 2016, 10:30)



* Конференция проводится при поддержке гранта РФФИ 16-01-20629 Г

We apply the holographic approach to photosynthesis that is an important example of nontrivial quantum phenomena relevant for life which is being studied in the emerging field of quantum biology. We use the holographic approach to evaluate the time dependence of entanglement entropy and quantum mutual information in the Fenna- Matthews-Olson (FMO) light-harvesting complex in bacteria during the transfer of an excitation from an antenna to a reaction center. We show that the time evolution of the mutual information simulating the GKS-L open quantum systems master equation for the FMO complex in some cases can be obtained by means of a dual gravity describing black hole formation in the AdS-Vaidya spacetime.

The talk is based on the joint work with I. Arefeva, arXiv:1603.09107v2.

## Dark states and supertransport in quantum photosynthesis

Sergei Kozyrev
*Steklov Mathematical Institute*

(October 3, 2016, 11:00)

We consider a model of quantum photosynthesis with the following properties: Degeneracy in the light-harvesting system, Excitons in chromophores interact with three quantum fields (photons, phonons, sink). We show that degeneracy may lead to the supertransport effect (coherent amplification of the transport). Interaction with photons and phonons in our model is given by non-parallel vectors of bright states. Therefore bright photonic states can be dark phononic states. This leads to generation of non-decaying dark phononic states. We discuss the relation to experiments with photonic echo in light harvesting systems (this effect of existence of photonic echo is known as quantum photosynthesis).

## Adaptive dynamics and its application for quantum algorithm

Satoshi Iriyama
*Tokyo University of Science*

(October 3, 2016, 12:00)

Ohya proposed the Adaptive Dynamics which is a new mathematical treatment of complex systems, for instance, chaos, bio-information, psychological

phenomena, entanglement in quantum physics, and so on. We have studied on quantum algorithm for several years, and developed efficient algorithm for NP-complete problems. In this talk, we explain the mathematical framework of adaptive dynamics and recent result on quantum algorithm.

## Maslov's canonical operator on a pair of Lagrangian manifolds and Green function type asymptotics

Sergey Dobrokhotov
*Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow*

(October 3, 2016, 12:30)

Maslov's canonical operator on a pair of Lagrangian manifolds and Green function type asymptotics

We consider differential and pseudodifferential equations with properties similar to those of the Helmholtz equation with spatially localized right hand side (in particular, the Dirac delta function). Our analysis uses ideas close to ones due to Keller, Babich, Kucherenko, Melrose, Uhlmann, Sternin, and Shatalov and shows that an asymptotic solution of such an equation can be represented via Maslov's canonical operator on an appropiate pair of Lagrangian manifolds. By way of example, a pseudodifferential equation arising in linear water wave theory is presented.

This work was done together with A.Anikin, V.Nazaikinskii, and M.Rouleux and was supported by RFBR grant 14-01-00521.

## Propagation of localized quantum states on singular manifolds.

Andrei Shafarevich
*Moscow State University*

(October 3, 2016, 13:00)

We study propagation of localized quantum states on singular spaces, obtained by gluing several Riemannian manifolds by a number of segments. We describe the relation of this quantumproblem to the behaviour of geodesics on the manifolds as well as to the certain problems of analytic number theory.

## Quantization of Sobolev space of half-differentiable functions

Armen Sergeev
*Steklov Mathematical Institute*

(October 3, 2016, 15:00)

We shall construct the quantization of the Sobolev space $V = H_0^{1/2}(S^1, \mathbb{R})$ of half-differentiable functions on the circle closely related to the string theory. The group $\mathrm{QS}(S^1)$ of quasisymmetric homeomorphisms of the circle acts on this space by reparameterization but this action is not smooth. However, we can introduce a quantized infinitesimal action of $\mathrm{QS}(S^1)$ on the Sobolev space $V$ which allows us to construct a quantum algebra of observables associated with the classical system $(V, \mathrm{QS}(S^1))$.

## Quantization triggered by breaking commutativity of symmetries. Discrete spectrum and tunnelling transfer controlled by distortion.

Mikhail Karasev
*National Research University "Higher School of Economics" (HSE), Moscow*

(October 3, 2016, 15:30)

By an important physical example of electron moving in a planar Penning trap we explain the general role of resonances making the symmetry algebra of a system to be noncommutative. After tuning at resonance the classical system can obtain quantum behavior, for instance, well distinguishable energy levels and opportunities for tunnelling transfer. Quantum properties are controlled by distortions in geometry of the system and by anharmonic parts of potentials. These controlling instruments determine the effective Hamiltonian over the noncommutative, non-Lie symmetry algebra which is the very object subjected to quantization. Tuning at resonance creates a (gyroscopic) symplectic structure as a mechanism for appearing additional forces, calls on a new (gyron) Hamiltonian and hence generates a specific mechanics in the space of symmetries. This mechanics can become quantum although the particle itself (core) remains to be classical. The point is that the dynamics in the gyroscopic structure is decelerated thus making the de Broglie wavelength much bigger in directions transverse to the classical trajectory and critically increasing the probability of quantum translocations of the particle.

## The binary represectation of quantum observables

Mikhail Ivanov
*Moscow Institute of Physics and Technology*

(October 3, 2016, 16:00)

To model a quantum system on quantum computer one needs to represent continuous observables (coordinates, momenta, etc.) in terms of binary observables. The problem is considered in terms of representation of observables by series of powers of 2, similar to binary representation of real numbers. The coefficients of series ("digits") are orthogonal projector operators. The representations provides a natural method of renormalization of infinite series and integrals.

## Renormalization group dynamics in the plane of the coupling constants of the fermionic hierarchical model

Mukadas Missarov, A. F. Shamsutdinov
*Kazan Federal University, Kazan Federal University*

(October 3, 2016, 17:00)

We consider four-component fermionic (Grassmann-valued) field on the hierarchical lattice. The Gaussian part of the Hamiltonian in the model is invariant under the block-spin renormalization group transformation with given degree of normalization factor ( renormalization group parameter). The non-Gaussian part of the Hamiltonian is given by the self-interaction forms of the 2-nd and 4-th order with coupling constants $r$ and $g$. The action of the renormalization group transformation in the model is reduced to the rational map in the plane of coupling constants $(r, g)$. The upper half-plane $\{(r, g) : g > 0\}$ and the lower half-plane are invariant under the renormalization group transformation We investigate the dynamics of this map in the lower and upper half-planes for different values of renormalization group parameter. To describe the global picture of the renormalization group flow we use also the space of the coefficients of the expansion of free measure density wich is denoted as the c-space. The renormalization group action in c-space is given as a homogeneous quadratic map. This space is treated as a two-dimensional projective space and is visualized as a unit disk. If the renormalization group parameter is greater than the dimension of the lattice, then the only attracting fixed point of the renormalization group transformation is defined by the density of the Grassmann delta-function . We describe two different (left and right) invariant neighborhoods of this fixed point and classify the

points on the plane according to the way they tend to this fixed point (from the left or from the right). We describe explicitly the zone structure of the classified domains and show that the global renormalization group flow has a nice description in terms of this zone structure. We discuss also the global behavior of all RG-invariant curves.

## Dynamics of a mobile impurity in a one-dimensional quantum fluid

Oleg Lychkovskiy
*International Center for Quantum Optics and Quantum Technologies (the Russian Quantum Center), Steklov Mathematical Institute*

(October 3, 2016, 17:30)

Consider a mobile impurity particle injected in a one-dimensional quantum fluid with some initial velocity, $v_0$. What will be the relaxation dynamics of the impurity? Numerical and seminumerical studies of finite systems ($N < 50$, where $N$ is the number of particles of the fluid) revealed a highly nontrivial dynamics: The impurity's velocity experienced oscillations superimposed on a slowdown; finally the velocity apparently saturated at some non-zero value, $v_f$ [1,2]. These studies, while producing much excitement, left unanswered basic questions on the nature of the effects discovered. It was even unclear whether the the incomplete relaxation was a finite-size effect or an effect present in the thermodynamic limit.

We present a detailed analytical study of the anomalous relaxation dynamics of an impurity particle injected in the one-dimensional quantum fluid [3-7]. In particular, we rigorously prove that the impurity particle of finite mass never stops completely, even in the thermodynamic limit [3,4]. This should be contrasted with the well-known absence of superfluidity in one dimension. These two facts can be reconciled since $v_f$ depends on the mass of the particle and vanishes for the infinite mass, which is equivalent to the absence of superfluid flow through a static constriction. We also find analytical dependence of the final velocity, $v_f$, on the initial velocity, $v_0$, for particular quantum fluids, the one-dimensional Fermi gas and the gas of impenetrable bosons [5-7].

## Список литературы

[1] C. J. M. Mathy, M. B. Zvonarev, and E. Demler, "Quantum flutter of supersonic particles in one-dimensional quantum liquids", *Nature Physics*, **8** (2012), 881–886 crossref WEB OF SCIENCE™ Scopus.

[2] M. Knap, C. J. M. Mathy, M. Ganahl, M. B. Zvonarev, and E. Demler, "Quantum flutter: Signatures and robustness", *Phys. Rev. Lett.*, **112** (2014), 015302 crossref WEB OF SCIENCE™ Scopus.

[3] O. Lychkovskiy, "Perpetual motion of a mobile impurity in a one-dimensional quantum gas", *Phys. Rev. A*, **89** (2014), 033619 crossref WEB OF SCIENCE™ Scopus.

[4] O. Lychkovskiy, "Perpetual motion and driven dynamics of a mobile impurity in a quantum fluid", (Rapid Communication), *Phys. Rev. A*, **91** (2015), 040101 crossref Scopus.

[5] E. Burovski, V. Cheianov, O. Gamayun, and O. Lychkovskiy, "Momentum relaxation of a mobile impurity in a one-dimensional quantum gas", (Rapid Communication), *Phys. Rev. A*, **89** (2014), 041601 crossref Scopus.

[6] O. Gamayun, O. Lychkovskiy, V. Cheianov, "Kinetic theory for a mobile impurity in a degenerate Tonks-Girardeau gas", *Phys. Rev. E*, **90** (2014), 032132 crossref WEB OF SCIENCE™ Scopus.

[7] O. Gamayun, E. Burovski, V. Cheianov, O. Lychkovskiy, M. Malcomson, M. Zvonarev, in preparation.

## Field-particle structures explicitly defined by any shear-free null congruence

Vladimir Kassandrov

*Institute of Gravitation and Cosmology*

(October 3, 2016, 17:50)

Celebrated Kerr theorem pus in correspondence twistor functions in $CP^3$ and shear-free null congruences in M. We demonstrate that any twistor function algebraically defines also a whole set of relativistic vacuum fields – Maxwell, Yang-Mills, Weyl and eikonal ones, together with a Kerr-Shild metric. Common singularities of these fields possess some properties of quantum particles (e.g., the integer-valued electric charges) and are involved in a nontrivial "algebraic" dynamics. The latter turns out to be conservative for a wide class of holomorphic generating twistor funcions.

## An empirical study on the common approach shared by the frequentist and subjective authors in the probability theory

Paolo Rocchi

*IBM & LUISS University*

(October 3, 2016, 18:10)

We have conducted a textual analysis on the most significant work written by eight masters of the probability theory: Kolmogorov, von Mises, de Finetti, Savage and others. The search consists in the systematic analysis of the content, the structure and the functions of the central messages contained in texts. The inquiry brings evidence how frequentist and subjective authors begin their studies with a similar style. Each one discusses the various viewpoints; he selects the interpretation of probability that he considers to be the best and erects a theory on the basis of his choice. The masters reach different conclusions but share a common premise that appears to be strongly characterized by philosophical topics and personal criteria. We call discretionary approach, the way taken by the frequentist and the subjectivist masters, and we remark how they overlook the standard method – a typical part of the science theorization – which every physicist, engineer, economist and other scientists adopt whenever they mean to establish a new measure. This universal method is based on the formal definition of the intended measure and all that experts need is derived from that mathematical definition. Why should the probability foundations disregard the standard method?

## A Dual version of Narsimhan-Seshadri Theorem in Seiberg-Witten Gauge Theories.

Gourab Bhattacharya
*Information Retrieval Society of India*

(October 3, 2016, 18:30)

As Narsimhan-Seshadri Theorem can be proved via SU(2)- (Anti) Self-dual Yang-Mills Gauge Theory over Riemann Surfaces via methods of Donaldson, we tried to understand the status of the theorem in other ”dual” (the term “dual” will be specified in the lecture as a certain kind of Langlands duality.) Gauge Theories, specifically to (may be Non abelian) Seiberg-Witten Gauge Theories, and in doing so we study a specific kind of Moduli Problem, namely “Moduli of Higgs Bundles "arising from N = 2 Twisted Supersymmetry of Witten.

## p-Adic Aspects of Gravity and Cosmology

Branko Dragovich
*Institute of Physics, Belgrade, University of Belgrade, Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrad*

(October 4, 2016, 10:30)

We consider possible modification of General Relativity inspired by p-adic string theory. The corresponding consequences on modern cosmology will be discussed.

## Cosmological models with non-positive definite potentials and bounce solutions

Sergey Vernov
*Lomonosov Moscow State University, Skobeltsyn Institute of Nuclear Physics*

(October 4, 2016, 12:00)

In the General Relativity cosmological models the initial period of the Universe evolution with energies above the Planck scale should be described by quantum gravity because the classical evolution includes the initial singularity. Important question of theoretical cosmology is whether the entire Universe evolution can remain classical and has no singularity. Modified gravity models of bouncing universes with a period of contraction followed by a bounce and a resent period of expansion attract a lot of attention. At the bounce point the Hubble parameter is equal to zero, therefore, the potential of the scalar field minimally or non-minimally coupled to gravity should be negative. The dynamics of non-minimally coupled scalar field cosmological models with non-positive definite potential has been studied by I.Ya. Aref'eva, N.V. Bulatov, R.V. Gorbachev, and S.Yu. Vernov (Class. Quantum Grav. 31 (2014) 065007). Using the results of this paper, we consider cosmological models including the Hilbert-Einstein curvature term and the induced gravity term with a negative coupled constant. The considering models generalize the integrable cosmological model with bounce solutions proposed by B. Boisseau, H. Giacomini, D. Polarski, and A.A.Starobinsky (J. Cosmol. Astropart. Phys. 1507 (2015) 002). The case when the scalar field has the conformal coupling and the Higgs-like potential with an opposite sign is studied in detail. We show that in the proposed model the evolution of the Hubble parameter of the bounce solutions can be non-monotonic and essentially depends on the sign of the constant term in the potential.

## Regular and chaotic regimes in scalar field FRW cosmology

Alexey Toporensky
*Lomonosov Moscow State University, P. K. Sternberg Astronomical Institute*

(October 4, 2016, 12:20)

The cosmological dynamics of a closed isotropic Universe filled by a scalar field is considered. We describe three different global regimes which exist in this model. Two of them include a chaotic behavior. Using Poncare section method we show that one of these regimes containes islands of quasiperiodic trajectories.

## Inflation from non-minimal coupled Higgs sector

Ekaterina Pozdeeva
*Lomonosov Moscow State University, Skobeltsyn Institute of Nuclear Physics*

(October 4, 2016, 12:40)

We consider the inflation scenarios in non-minimally coupled models with potentials corresponding to the Higgs sector of the Grant Unification Theory. The obtained inflation models are in good agreement with the most recent and accurate observational data.

## Observational constraints on graviton mass

Alexander Zakharov
*Institute of Theoretical and Experimental Physics*

(October 4, 2016, 13:00)

In the papers reporting about this discovery, the joint LIGO & VIRGO team presented an upper limit on graviton mass such as $m_g < 1.2 \times 10^{-22} eV$ (Abbott et al. 2016). The authors concluded that their observational data do not show violations of classical general relativity because the graviton mass limit is very small. We show that an analysis of bright star trajectories could bound graviton mass with a comparable accuracy with accuracies reached with gravitational wave interferometers and expected with forthcoming pulsar timing observations for gravitational wave detection. This analysis gives an opportunity to treat observations of bright stars near the Galactic Center as a tool for an evaluation specific parameters of the black hole and also to obtain constraints on the fundamental gravity law such as a modifications of Newton gravity law in a weak field approximation. In that way, based on a potential reconstruction at the Galactic Center we give bounds on a graviton mass.

## Supersymmetric bag model as a new path to particle physics compatible to gravity

Alexander Burinskii
*Nuclear Safety Institute, RAS, Moscow*

(October 4, 2016, 13:20)

We show that large spin of particles does gravity strong, which shifts the usual Planck scale to effective action at the Compton distances. Considering Kerr's spinning particle with parameters of electron, we obtain a BPS solution for the core of spinning particle as a supersymmetric bag model, a gravitating analog of the known bag models. The shape of bag is deformed by spin to a disk which acquires a closed string and singular pole on the border of bag. The supersymmetric Kerr's bag model resolves the problem of compatibility gravity with quantum theory, forming a 4D analog of the D2-D1-D0-brane system of string–M-theory without extra dimensions and compactification.

## Список литературы

[1] A. Burinskii, "Gravitating Lepton Bag Model", *JETP*, **148**:2 (2015), 228; "Stability of the lepton bag model based on the Kerr-Newman solution", № 5, 937.

## Constructive Combinatorial Field Theory

Vincent Rivasseau
*Université Paris-Sud, Orsay cedex*

(October 4, 2016, 15:30)

Combinatorial Field Theory studies quantum field models which are independent of any particular space-time background, such as vector, matrix and tensor models. A main motivation is to develop a background independent formalism for quantizing gravity. We shall review recent progress which allows to build several such combinatorial models at the non-perturbative constructive level.

## Quantum anomalies and transformations of Feynman path integrals

Oleg Smolyanov
*Lomonosov Moscow State University*

(October 4, 2016, 16:00)

One intends to show that both explanation of the origin of quantum anomalies and Noether's theorems can be obtained using similar formulas for derivatives of functions whose values are either measures or pseudomeasures. By this way a contradiction between explanations of quantum anomalies given by Brice DeWitt, on one hand, and by Fujikawa and Suzuki, on another hand, is resolved.

## Symmetry breaking effective potentials extreme properties for the standard model extensions

Mikhail Dolgopolov
*Samara State University*

(October 4, 2016, 17:30)

The evaluation and systematization of the one-loop (and multi-loop) corrections to the control parameters of effective Higgs potential at finite temperature, the determination of the phase transitions and constrains combined with experimental ones of the Large Hadron Collider and cosmology will be presented.

## On uniqueness problems related to continuity and Fokker-Planck-Kolmogorov equations

Stanislav Shaposhnikov
*Lomonosov Moscow State University*

(October 4, 2016, 18:10)

We present a new approach to specify a class of measures which, under reasonable assumptions about coefficients and initial data, there is a unique solution to the Cauchy problem. It turns out that even in the one-dimensional case in the present framework new results can be obtained. Moreover we present some new interesting applications to infinite dimensional equations.

## Cluster tau-functions and quantum generating series

Dimitry Noshchenko
*Institute of Cosmophysical Researches and Radio Wave Propagation, Far East Division, Russian Academy of Sciences, Kamchatka State University named after Vitus Bering*

(October 4, 2016, 18:30)

We introduce a class of "tau-functions" associated with quiver representations. Our claim is that certain generating series (in the context of quantum topology) can be described in terms of such functions.

## Chern-Simons term in the geometric theory of defects

Mikhail Katanaev, J. Zanelli
*Steklov Mathematical Institute, Centro de Estudios Científicos*

(October 5, 2016, 12:00)

The Chern-Simons term is used in the geometric theory of defects. The equilibrium equations with $\delta$-function source are explicitly solved with respect to the SO(3) connection. This solution describes one straight linear disclination and corresponds to the new kind of geometrical defect: it is the defect in the connection but not the metric which is the flat Euclidean metric. This is the first example of a disclination described within the geometric theory of defects. The corresponding angular rotation field is computed.

## Stability of Quantum Statistical Ensembles with Respect to Local Measurements

Boris Fine
*Skolkovo Institute of Science and Technology*

(October 5, 2016, 12:30)

We introduce a stability criterion for quantum statistical ensembles describing macroscopic systems. An ensemble is called "stable" when a small number of local measurements cannot significantly modify the probability distribution of the total energy of the system. We apply this criterion to lattices of spins-1/2, thereby showing that the canonical ensemble is nearly stable, whereas statistical ensembles with much broader energy distributions are not stable. In the context of the foundations of quantum statistical physics, this result justifies the use of statistical ensembles with narrow energy distributions such as canonical or microcanonical ensembles.

Reference: arXiv:1601.06402

## Holographic thermalization from colliding particles

Mikhail Khramtsov
*Steklov Mathematical Institute*

(October 5, 2016, 13:00)

The talk is devoted to the study of thermalization process in the boundary theory in AdS3/CFT2 holographic setting. We discuss the dynamical picture of black hole formation in the bulk and describe it holographically by analyzing the behavior of correlation functions in the boundary theory. We observe that the calcuulation of two-point correlators in geodesic approximation reproduces thermal behavior if winding geodsics are taken into account.

## Note on Entropy for Quantum Open Systems

Noboru Watanabe
*Tokyo University of Science*

(October 6, 2016, 10:00)

In order to treat several complicated systems, it is important to study the dynamics of state change and the complexity of states of systems. Information Dynamics introduced by Ohya is a new concept synthesizing the research schemes of several complicated systems. In ID, there are two types of complexities, (1) a complexity of state describing system itself and (2) a transmitted complexity between two systems. Classical and quantum entropies are the example of the complexities of those complexites. Based on the Umegaki's relative entropy of Umegaki and the compound state, the quantum mutual entropy was defined by Ohya in 1983, and it was extended to general quantum systems by using the relative entropy of Araki and Uhlmann. One can discuss the coding theorems by means of the mean entropy and the mean mutual entropy defined by the dynamical entropy. In this talk, we will discuss about entropy of open system dynamics.

## Quantum "Signal plus stationary coloured noise" models

Alexander Holevo
*Steklov Mathematical Institute of Russian Academy of Sciences, Moscow*

(October 6, 2016, 10:30)

Quantum Gaussian "signal plus noise" models with stationary coloured noise are considered in the framework of quantum Shannon theory. A proof is given of the coding theorem for the classical capacity of a quantum channel with broadband stationary noise.

## Conditional mutual information in infinite-dimensional quantum systems and its use

Maksim Shirokov
*Steklov Mathematical Institute*

(October 6, 2016, 11:00)

It is shown that the quantum conditional mutual information (defined in a standard way) has a unique lower semicontinuous extension to the set of all infinite-dimensional tripartite states. Some corollaries of the lower semicontinuity of the conditional mutual information are discussed. Applications to the theory of infinite-dimensional quantum channels and their capacities are also considered.

## On the definition of entropy production in Markovian open quantum systems

Anton Trushechkin
*Steklov Mathematical Institute*

(October 6, 2016, 12:00)

Entropy production for open quantum systems is defined as the sum of the increment of the entropy of the system and the entropy flow from the system into the environment. However, the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation does not provide information about the environment and how it is coupled to the system. So, the entropy flow to the environment and, hence, the entropy production cannot be determined from the GKSL equation alone. We address the following question: what minimal additional information should be supplemented to the GKSL equation that is sufficient to determine the entropy production. We approach this problem using the concept of complementary quantum channel from quantum information theory.

## On the asymptotical normality for lattice Hamiltonian dynamics. Energy transport equation

Tatiana Dudnikova
*Keldysh Institute of Applied Mathematics of Russian Academy of Sciences, Moscow*

(October 6, 2016, 12:30)

We consider the lattice dynamics in the whole space (in the half-space) and study the Cauchy (respectively, mixed initial-boundary value) problem

with random initial data. We prove the weak convergence of statistical solutions to a limit for large time. Further, we assume that the initial measure enforces slow spatial variation on the linear scale $1/\varepsilon$. We check that for times of order $1/\varepsilon$, the limit covariance changes in time and is governed by the energy transport equation.

## An example of quantum system control with coherent feed-back

Viktoryia Dubravina
*Lomonosov Moscow State University*

(October 6, 2016, 13:00)

There exist two specific ways of quantum system control, namely, the one described by Schrodinger equation and the one, described the reduce of state as a result of some quantum measurement. The case when no measurements acquire that can destroy coherence is said to be quantum control with coherent feed-back or coherent quantum control, otherwise it is said to be non-coherent case.

A quantum system consisting of one free electron is taken as en example. A pure state of such system can be described by en element of the space $H = L_2(\mathbb{R}^3) \times \mathbb{C}^2$, and the mixed state is represented by a linear trace-class positively-defined self-adjoint operator from $L_1(H)$ with unitary trace. Let its spin originally has a known pure state $A \in H$. This means that upon a measurement the state projection on some fixed direction $h_A \in \mathbb{R}^3$ equals 1 with full probability. As a result of this quantum system control we would like to obtain the electron with spin in pure state $B$, such that upon a measurement the state projection on orthogonal to $h_A \in \mathbb{R}^3$ direction $h_B$ equals 1 with full probability.

The example of control under consideration exploits Stern–Gerlach experiment concept. We pose a cascade of magnetic fields so that every proceeding cascade element partially effects the system state. For large enough number for magnetic fields investigated construction provides quantum system state close enough to desired one.

## On the law of large numbers for the composition of random semigroups

Vsevolod Sakbaev
*Moscow Institute of Physics and Technology*

(October 6, 2016, 15:00)

The random linear operators in Banach spaces and one-parameter semigroups of such operators are studied. The low of large numbers for the sequence of compositions of independent random operators is investigated. The sufficient conditions and the examples of violation for the low of large numbers for the sequence of compositions of undependent random semigroups of linear operators in Hilbert space are obtained.

## On the one-dimensional continuity equation with a nearly incompressible vector field

Nikolay Gusev
*Steklov Mathematical Institute*

(October 6, 2016, 15:30)

We consider the Cauchy problem for the continuity equation with a bounded nearly incompressible vector field $b\colon (0,T)\times\mathbb{R}^d \to \mathbb{R}^d$, $T > 0$. This class of vector fields arises in the context of hyperbolic conservation laws, in particular in connection with the Keyfitz-Kranzer system.

It is well known that in the generic multi-dimensional case ($d \geq 1$) near incompressibility is sufficient for existence of bounded weak solutions, but uniqueness may fail (even when the vector field is divergence-free), and hence further assumptions on the regularity of $b$ (e.g. Sobolev regularity) are needed in order to obtain uniqueness.

We prove that in the one-dimensional case ($d = 1$) near incompressibility is sufficient for existence and uniqueness of locally integrable weak solutions. We also study compactness properties of the associated Lagrangian flows.

## On uniqueness of weak solutions to transport equation with non-smooth velocity field

Paolo Bonicatto
*International School for Advanced Studies (SISSA)*

(October 6, 2016, 15:50)

Given a bounded, autonomous vector field $b\colon \mathbb{R}^d \to \mathbb{R}^d$, we study the uniqueness of bounded solutions to the initial value problem for the associated transport equation

$$\partial_t u + b\cdot\nabla u = 0. \tag{1}$$ {eq:transport}

This problem is related to a conjecture made by A. Bressan, raised studying the well-posedness of a class of hyperbolic conservation laws. Furthermore,

from the Lagrangian point of view, this gives insights on the structure of the flow of non-smooth vector fields.

In the talk we will discuss the two dimensional case and we prove that, if $d = 2$, uniqueness of weak solutions for (**??**) holds under the assumptions that $b$ is of class BV and it is *nearly incompressible*. Our proof is based on a splitting technique (introduced previously by Alberti, Bianchini and Crippa) that allows to reduce (**??**) to a family of 1-dimensional equations which can be solved explicitly, thus yielding uniqueness for the original problem. This is joint work with S. Bianchini and N.A. Gusev.

## On the Morse–Sard theorem for the sharp case of Sobolev mappings and its applications in fluid mechanics

Mikhail Korobkov
*Sobolev Institute of Mathematics*

(October 6, 2016, 16:10)

We establish Luzin N- and Morse–Sard properties for the sharp case of Sobolev–Lorentz classes $W^k_p(R^n, R^m)$ under minimal integrability assumptions (that garantee the continuity of a mapping only, i.e., $p = n/k$). Using these results we prove that almost all level sets of such functions are finite disjoint unions of $C^1$–smooth compact manifolds of dimension $n-m$ (despite the fact that a function itself is not $C^1$ — it is continuous only).

These results helped in mathematical fluid mechanics — for the so-called Leray's problem, which remained open for more than 80 years (starting from the publication of the famous paper of Jean Leray 1933 ). Namely, for plane and axially symmetric spatial flows the existence theorem was proved for boundary value problem of stationary Navier-Stokes equations in bounded domains under necessary and sufficient condition of zero total flux.

## On the structure of non-commutative operator graphs

Grigori Amosov
*Steklov Mathematical Institute*

(October 6, 2016, 17:00)

Some operator graphs associated with quantum channels can be obtained as non-commutative deformations of the algebra generated by the Klein

group. The critical point in this deformation gives rise to the superactivation of a channel.

## Thermal preparation of an entangled steady state of spin ensembles

Natalia Teper
*Ul'yanovsk State Pedagogical University*

(October 6, 2016, 17:50)

Steady state entanglement is studied in the continuous-variable system of two distant ensembles of electronic spins driven by squeezed microwave field. Proposed circuit consists of two distant nitrogen-vacancy-center ensembles coupled to transmission line resonator. Squeezed microwave field is generated by a flux-driven Josephson parametric amplifier. The time evolution of the system is described by the master equation for density operator. Criterion based on logarithmic negativity is used for the detection of entanglement. Squeezed field of the Josephson parametric amplifier is driving force, which provides steady state entanglement between two bosonic modes of nitrogen-vacancy-center ensembles for a wide range of system parameters. This can open the way for implementation of continuous-variable states for quantum computing applications.

## Kinetics of CO connection using mioglobin: rigorous analytic results and predictions

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(October 7, 2016, 11:00)

В экспериментах по исследованию динамики и функций белка особое место занимает кинетика связывания $CO$ миоглобином [**?**, **?**]. Эти эксперименты были проведены на большой шкале временных масштабах ($10^{-7} - 10^{2}$ с) и в широком диапазоне температур ($60 - 300$ K). В этих экспериментах сразу обращает на себя внимание тот факт, что в высокотемпературной ($200 - 300$ K) и низкотемпературной ($60 - 190$ K) областях кинетика связывания существенно различаются. В одном и том же временном окне наблюдения кинетические кривые можно аппроксимировать степенными законами. При этом в высокотемпературной области на

степенном участке кривой кинетика связывания описывается аномальной температурной зависимостью. В низкотемпературной области в том же окне наблюдения кинетика описывается также степенной функцией, но с нормальной температурной зависимостью. Другими словами имеет место скачкообразное изменение показателей степенных аппроксимаций в узкой температурной зоне от 180 до 200 K, меняющее температурную зависимость скорости реакций на прямо противоположную. Этот факт как иыяснилось вызывает основное затруднение в построении теории, описывающей единым образом кинетику связывания $CO$ миоглобином во всем диапазоне температур. В нашей обзорной статье [?] дан анализ наиболее разумных попыток построения таких теорий. В этой же работе была предложена ультраметрическая модель кинетики связывания $CO$, которая была исследована численно. Как оказалось данная модель правильно воспроизводит температурную зависимость показателя степени кинетических кривых во всем диапазоне температур от 60 до 300 K. В данном докладе приводим математически строгое описание модели, предложенной в [?]. Мы аналитически находим зависимость показателей степеней кинетических кривых от температуры как в высокотемпературной, так и в низкотемпературной областях. Мы также обсуждаем важную особенность рассматриваемой модели, которая заключается в том, что при увеличении времени наблюдения на низкотемпературных кинетических кривых появляется второй степенной участок, который имеет аномальную температурную зависимость. Возможно, что этот факт может служить основанием для предсказания будущих экспериментов по кинетики связывания белков с лигандами.

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## From geometric optics to plants: eikonal equation for buckling

Kirill Polovnikov
*Physics Department, Moscow State University*

(October 7, 2016, 11:20)

Optimal isometric embedding to three-dimensional space of exponentially growing squeezed 2D surfaces, like plant leaves, lilies or other colonies of exponentially reproducing cells, is considered in the framework of conformal approach. It is shown that a boundary profile, adopted by the growing tissue is described by the eikonal equation, which provides the geometric optic approximation for the wave front propagating in the media with inhomogeneous refraction coefficient. The detailed spatial dependence of the refraction coefficient is connected with the specific form of conformal mapping which in turn is dictated by the local growth protocol. We show that numerical patterns predicted by the eikonal equation and development of buckling instabilities are strikingly similar with those found in nature.

## On the one-particle formulation of the Dirac theory

Nikolay Chuprikov
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(October 7, 2016, 12:00)

Dirac's theory is presented as a logically consistent four-component quantum theory of a relativistic particle with the half-integer spin. In this approach the complete system of independent solutions to the problem for a free Dirac particle with a definite energy and momentum, collinear to a given direction, contains four solutions for each sign of energy, rather than two, as in the standard approach. As was shown, solving the Dirac equation for a particle with a definite energy is reduced either to solving the generalized Pauli equation for a "heavy" quasiparticle or to solving the generalized Pauli equation for a "light" quasiparticle; the effective mass of each quasiparticle carriers information about the symmetry of the four-dimensional space-time. But Dirac's theory is irreducible to the two-component Pauli theory even in the non-relativistic limit. Unlike the latter the former describes a particle with two internal degrees of freedom, rather than one. The second internal degree of freedom of the Dirac particles is associated with the intrinsic parity, with the (conditional) positive intrinsic parity being associated with the "heavy" quasiparticle and (conditional) negative intrinsic parity being associated with the "light" quasiparticle. "Physical states" of the Dirac particle always represent mixtures of states with positive and negative internal parities. For a particle

with a definite energy and momentum the ratios of the effective masses to their sum give the probabilities of finding the Dirac particle in the corresponding subensembles of quasiparticles. The "formal states" of the Dirac particle with a negative energy should be considered as "physical states" of its antiparticle with a positive energy: no superposition of states of the Dirac particle with different signs of energy is allowed. The space of states with positive energies is closed with respect to the action of even and odd operators.

## Towards causality formalism of quantum mechanics

Vladimir Anashin
*Lomonosov Moscow State University*

(October 7, 2016, 12:30)

Transducers (in particular, Mealy automata) constitute a formal model for causality and discreteness. It will be shown that Hilbert space-based QM formalism can be (to some extend) deduced mathematically from that model. Interestingly, the model leads to finite-dimensional (rather to infinite-dimensional) Hilbert spaces, but of very high dimension about $10^{45}$ (actually of order $\ln 2/\tau$, where $\tau$ is Planck time). The model also gives some evidence that in $p$-adic QM it should be taken $p = 2$.

## The hierarchy in the natural protein sequences and it's applications

Anastasia Anashkina
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(October 7, 2016, 13:00)

A method was proposed for the analysis of protein sequences. The method allows identifying the hierarchical organization of structural information of protein sequences. Research of the entropy characteristics of natural polypeptide sequences [1] reveals maximal size of protein sequence with minimal and constant entropy value as five adjacent amino acids. Such a group of five neighboring residues was used as a minimal unit of protein sequence and has been called the "information unit". Based on information units representation, special mathematical approach for protein sequence analysis was called "Analysis of Information Structure" method (ANIS method) [2]. ANIS method consists of several steps. Initially, the frequency of occurrence in the database of non-homologous protein sequences is determine for each of the information units

that comprise the studied protein sequence. Then, "population profile" is built based on the frequency of occurrence of information units. On the last step "population profile" is decomposed into positive defined symmetric functions with different half-width. Hierarchically organized information structure of the protein sequence is a result of the ANIS method. The ANIS method enabling to reveal the hierarchical organization of structural information contained in amino acid sequence of proteins. The approach was tested in the set of experimental studies by protein engineering. For example, functionally important fragments of heat shock protein (hHSP70), human tumor necrosis factor (hTNF) and protein gp181 from phage KZ were obtained. The proposed approach can be used for de novo protein design. Applications of ANIS method in protein engineering [3, 4], studies of structural organization of enzymes [5, 6] and protein-protein complexes [7] are described. In general method described in [8, 9].

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## Measure-preserving and ergodic an n-unit delay mappings


Livat Tyapaev

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(October 7, 2016, 13:20)

The automaton transformation of infinite words over alphabet $F_p = 0, 1, .., p-1$, where $p$ prime number coincide with continuous transformation of a ring of p-adic integers $Z_p$. The object of this study is the an $n$-unit delay maps (associated with asynchronous automata) that is important for cryptography. We prove criteria of measure-preserving for an n-unit delay mappings. Moreover, we give a sufficient condition of ergodicity of such mappings.

## Generalized Clifford algebras

Nikolay Marchuk
*Steklov Mathematical Institute*

(October 7, 2016, 15:00)

We consider generalized Clifford algebras with a special commutation rule that depend on a primitive root from identity. In these algebras a main set of (noncommutative) generators is completed with a set of commutative generators. In more details we consider generalized Clifford algebras of second order. In particular, we discuss an operator of Hermitian conjugation of elements of generalized Clifford algebras.

## Covariantly constant solutions of the Yang-Mills equations

Dmitry Shirokov
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(October 7, 2016, 15:20)

We consider a class of new field equations and present general solution of these equations. These equations are invariant under orthogonal coordinate transformations and invariant under gauge transformations, which depend on some Lie groups. We use some new geometrical objects - Clifford field vector and an algebra of h-forms which is a generalization of the Atiyah-Kahler algebra and the algebra of differential forms. With the use of new field equations we present a class of covariantly constant solutions of the Yang-Mills equations.

The reported study was funded by Russian Science Foundation (project RSF 14-11-00687, Steklov Mathematical Institute) and by RFBR according to the research project No. 16-31-00347 mol_a.

## Asymptotic solutions of the Cauchy problem for a wave equation with rapidly varying coefficients

Vladimir Nazaikinskii
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(October 7, 2016, 15:40)

For the wave equation in which the squared wave propagation velocity is a small rapidly oscillating perturbation of a slowly varying function, we consider the Cauchy problem with initial data localized in a small neighborhood of some point. Assuming that the perturbation lies in some algebra of averageable functions and the small parameters characterizing the localization of the initial data and the oscillation rate and amplitude of the perturbation are related by certain inequalities, we show that the leading term of the asymptotics of the solution can be obtained by the replacement of the velocity with its local average. We discuss classes of averageable functions, the relationship between our approach and other approaches to homogenization, and possible applications to models of tsunami wave propagation.

This work was done together with S. Dobrokhotov and B. Tirozzi and was supported by RFBR grant 14-01-00521 and by the CINFAI-RITMARE project (Italy).

## The quantum transition probabilities as a functional integrals in energy transitionaly and numerical calculations

Alexander Biryukov
*Samara State University*

(October 7, 2016, 16:00)

Quantum transitions probability represented as a path integral in energy state space of investigated multi-level quantum system. Using this approach we considered dynamics of nitrogen molecules which interacting with laser pulses. Our computer simulations indicate complex dependence between high states excitation probability and laser intensity.

## Quantum-mechanical analogies in some models of informational interactions

Alexander Lebedev
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(October 7, 2016, 17:00)

Представлены модели информационных взаимодействий, основанные на вероятностной концепции смыслов. Привлечение разработанного математического аппарата из квантовой механики позволило по-новому взглянуть на процессы динамики терминов в предметных областях. Гипотеза о волновом характере информации позволила рассмотреть явления её интерференции и дифракции. На основе модели были произведены некоторые количественные оценки.

## Analytic solution of the boundary Dirichlet Problems for the Poisson equations in the cases of carbon nanotubes an graphene nanoribbons

Kanstantsin Vasileuski
*Belarusian State University*

(October 7, 2016, 17:20)

Analytic solutions of the boundary Dirichlet Problems for the Poisson equations in the cases of carbon nanotubes an graphene nanoribbons which haven't studied earlier are obtained.

## General-kind differential-difference elliptic equations in half-plane: asymptotic properties of solutions

Andrey Muravnik
*"Sozvezdie", Peoples Friendship University of Russia, Moscow*

(October 7, 2016, 17:40)

The contemporary theory of nonlocal problems (actively developed by various researchers nowadays) and the general theory of functional-differential equations, i. e., equations containing other operators (apart from differential ones) acting on the unknown function, are closely related between each other. Actually, it is reasonably to talk about a unified research area of mathematical physics, specified by the following circumstance: unlike the classical theory of equations of mathematical physics, the equation (boundary-value condition) links the values of derivatives of the unknown function at

*different* points. This qualitative difference generates various novelties both in research results and in available research methods. Also, it opens application areas that are not available for the classical theory.

An important part of the above theory is the theory of partial differential-difference equations. To investigate them, one can apply operational methods because translation operators are Fourier multipliers. For the parabolic case, it is implemented by the author in *J. Math. Sci. (New York), 216 (2016), No 3, 345–496*. The elliptic case (for unbounded domains) is still an almost open research area.

In the present talk, the Dirichlet problem (with bounded continuous boundary-value functions) in the half-plane is investigated for the second-order differential-difference equation containing, apart from differential operators, translation operators with respect to the variable parallel to the boundary line (the so-called *nonlocal variable*). Those *nonlocal terns* are assumed to be restricted by the only condition: the real part of the symbol of the operator acting with respect to the nonlocal variable is bounded from below by a positive constant.

We investigate the solution of the specified problem, which is smooth outside the boundary line, and prove its *asymptotical closeness* to the unique bounded solution of the same Dirichlet problem for the partial *differential* equation obtained from the original differential-difference one by the following way: all translations are assigned to be equal to zero.

## Grassmann Structure on the Antisymmetric Fock Spaces with Unitary Fourier Supertransforms

Nikolaj Shamarov
*Lomonosov Moscow State University*

(October 7, 2016, 18:00)

In the antisymmetric Fock space, generated by a finite dimensional Hilbert space, we introduce a structure of a Grassmann algebra such that the Fourier supertransforms became unitary. Such structures are useful in a new interpretation of Pauli operators.